Core Connections: Course 3
Checkpoint Materials

Notes to Students (and their Teachers)

Students master different skills at different speeds. No two students learn exactly the same way at the same time. At some point you will be expected to perform certain skills accurately. Most of the Checkpoint problems incorporate skills that you should have been developing in grades 6 and 7. If you have not mastered these skills yet it does not mean that you will not be successful in this class. However, you may need to do some work outside of class to get caught up on them.

Starting in Chapter 1 and finishing in Chapter 10, there are 11 problems designed as Checkpoint problems. Each one is marked with an icon like the one above. After you do each of the Checkpoint problems, check your answers by referring to this section. If your answers are incorrect, you may need some extra practice to develop that skill. The practice sets are keyed to each of the Checkpoint problems in the textbook. Each has the topic clearly labeled, followed by the answers to the corresponding Checkpoint problem and then some completed examples. Next, the complete solution to the Checkpoint problem from the text is given, and there are more problems for you to practice with answers included.

Remember, looking is not the same as doing! You will never become good at any sport by just watching it, and in the same way, reading through the worked examples and understanding the steps is not the same as being able to do the problems yourself. How many of the extra practice problems do you need to try? That is really up to you. Remember that your goal is to be able to do similar problems on your own confidently and accurately. This is your responsibility. You should not expect your teacher to spend time in class going over the solutions to the Checkpoint problem sets. If you are not confident after reading the examples and trying the problems, you should get help outside of class time or talk to your teacher about working with a tutor.

Checkpoint Topics

1. Computations with Positive Fractions
2. Evaluating Expressions and Order of Operations
3. Computations with Positive and Negative Fractions
4. Unit Rates and Proportions
5. Solving Equations
6. Multiple Representations
7. Transformations
8. Solving Equations with Fractions (Fraction Busters)
9. Scatter Plots and Association
10. Laws of Exponents and Scientific Notation
11. Problems Involving Square Roots

Checkpoint Number 1

Problem 1-90

Computations with Positive Fractions

Answers to problem 1-90: a. $\frac{22}{15} = 1\frac{7}{15}$, b. $\frac{5}{8}$, c. $7\frac{1}{12}$, d. $1\frac{5}{8}$, e. 4, f. $\frac{7}{12}$

To add or subtract fractions a common denominator is needed. Once a common denominator is found, you either add or subtract the numerators. To add or subtract mixed numbers, you can either add or subtract their parts, or you can change the mixed numbers to fractions greater than one. To multiply fractions, first change any mixed numbers to fractions greater than one, then multiply the numerators and multiply the denominators. To divide fractions, multiply the first fraction by the reciprocal of the second.

Example 1: Compute the sum: $8\frac{3}{4} + 4\frac{2}{5}$

Solution: This addition example shows adding the whole number parts and the fraction parts separately. The answer is adjusted because the fraction part is greater than one.

$$\begin{aligned} 8\tfrac{3}{4} &= 8 + \tfrac{3}{4} \cdot \tfrac{5}{5} = 8\tfrac{15}{20} \\ +4\tfrac{2}{5} &= 4 + \tfrac{2}{5} \cdot \tfrac{4}{4} = +4\tfrac{8}{20} \\ & \qquad\qquad 12\tfrac{23}{20} = 13\tfrac{3}{20} \end{aligned}$$

Example 2: Compute the difference: $2\frac{1}{6} - 1\frac{4}{5}$

Solution: This subtraction example shows changing the mixed numbers to fractions greater than one and then computing in the usual way.

$$\begin{aligned} 2\tfrac{1}{6} - 1\tfrac{4}{5} &\Rightarrow \tfrac{13}{6} - \tfrac{9}{5} \\ &\Rightarrow \tfrac{13}{6} \cdot \tfrac{5}{5} - \tfrac{9}{5} \cdot \tfrac{6}{6} \\ &\Rightarrow \tfrac{65}{30} - \tfrac{54}{30} = \tfrac{11}{30} \end{aligned}$$

Example 3: Multiply $3\frac{1}{3} \cdot 2\frac{1}{2}$

Solution: $3\frac{1}{3} \cdot 2\frac{1}{2} = \frac{10}{3} \cdot \frac{5}{2} = \frac{10 \cdot 5}{3 \cdot 2} = \frac{5 \cdot \cancel{2} \cdot 5}{3 \cdot \cancel{2}} = \frac{25}{3} = 8\frac{1}{3}$

Example 4: Divide $1\frac{1}{3} \div 1\frac{1}{2}$

Solution: $1\frac{1}{3} \div 1\frac{1}{2} = \frac{4}{3} \div \frac{3}{2} = \frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9}$

Now we can go back and solve the original problem.

a. $\frac{2}{3}+\frac{4}{5}=\frac{2}{3}\cdot\frac{5}{5}+\frac{4}{5}\cdot\frac{3}{3}=\frac{10}{15}+\frac{12}{15}=\frac{22}{15}=1\frac{7}{15}$

b. $1-\frac{3}{8}=\frac{8}{8}-\frac{3}{8}=\frac{5}{8}$

c. $4\frac{1}{3}+2\frac{3}{4}=4+2+\frac{1}{3}+\frac{3}{4}=6+\frac{1}{3}\cdot\frac{4}{4}+\frac{3}{4}\cdot\frac{3}{3}=6+\frac{4}{12}+\frac{9}{12}=6+\frac{13}{12}=7\frac{1}{12}$

d. $5\frac{1}{2}-3\frac{7}{8}=\frac{11}{2}-\frac{31}{8}=\frac{11}{2}\cdot\frac{4}{4}-\frac{31}{8}=\frac{44}{8}-\frac{31}{8}=\frac{13}{8}=1\frac{5}{8}$

e. $5\frac{1}{3}\cdot\frac{3}{4}=\frac{16}{3}\cdot\frac{3}{4}=\frac{\cancel{4}\cdot 4\cdot\cancel{3}}{\cancel{3}\cdot\cancel{4}}=\frac{4}{1}=4$

f. $\frac{7}{8}\div 1\frac{1}{2}=\frac{7}{8}\div\frac{3}{2}=\frac{7}{8}\cdot\frac{2}{3}=\frac{14}{24}=\frac{7}{12}$

Here are some more to try. Compute each of the following problems

1. $\frac{2}{3}+\frac{7}{10}$
2. $\frac{4}{5}-\frac{1}{3}$
3. $2\frac{1}{4}+\frac{7}{8}$
4. $4\frac{2}{7}-1\frac{1}{14}$
5. $1\frac{1}{2}+3\frac{5}{6}$
6. $\frac{2}{9}\cdot\frac{7}{8}$
7. $2\frac{3}{4}\cdot\frac{1}{6}$
8. $5\div\frac{1}{4}$
9. $1\frac{2}{3}-\frac{5}{6}$
10. $\frac{6}{7}\div 1\frac{2}{7}$
11. $3\frac{2}{5}-\frac{1}{10}$
12. $2\frac{1}{5}\cdot 1\frac{3}{4}$
13. $\frac{5}{16}\cdot 1\frac{1}{3}$
14. $2\frac{1}{4}+1\frac{4}{5}$
15. $1\frac{5}{8}\div 2\frac{3}{5}$
16. $\frac{2}{9}+2\frac{1}{3}$
17. $\frac{1}{3}\div 4$
18. $5\frac{1}{3}-\frac{5}{6}$
19. $1\frac{7}{8}\cdot\frac{2}{3}$
20. $2\frac{1}{5}\div 1\frac{1}{3}$
21. $4\frac{3}{8}+15\frac{7}{12}$
22. $6\frac{7}{12}-2\frac{1}{6}$
23. $11\frac{1}{3}\cdot 1\frac{1}{2}$
24. $2\frac{2}{3}\div 3\frac{7}{8}$

Answers:

1. $\frac{41}{30} = 1\frac{11}{30}$	2. $\frac{7}{15}$
3. $3\frac{1}{8}$	4. $3\frac{3}{14}$
5. $5\frac{1}{3}$	6. $\frac{7}{36}$
7. $\frac{11}{24}$	8. 20
9. $\frac{5}{6}$	10. $\frac{2}{3}$
11. $\frac{33}{10} = 3\frac{3}{10}$	12. $\frac{77}{20} = 3\frac{17}{20}$
13. $\frac{5}{12}$	14. $4\frac{1}{20}$
15. $\frac{5}{8}$	16. $2\frac{5}{9}$
17. $\frac{1}{12}$	18. $\frac{9}{2} = 4\frac{1}{2}$
19. $\frac{5}{4} = 1\frac{1}{4}$	20. $\frac{33}{20} = 1\frac{13}{20}$
21. $19\frac{23}{24}$	22. $4\frac{5}{12}$
23. 17	24. $\frac{64}{93}$

Checkpoint Number 2

Problem 2-86

Evaluating Expressions and Order of Operations

Answers to problem 2-86: a. –8, b. 1, c. –2, d. 17, e. –45, f. 125

To evaluate an expression, calculate the value of the expression after each variable has been replaced with its numerical value. Be sure to follow the order of operations when doing the calculations as shown below:

Rules for Order of Operations (after substitution)

1. *Circle* the parenthesis or any other grouped numbers, and then circle the terms inside the grouping. (A fraction bar acts as parenthesis.)
2. *Simplify* each term in the parenthesis.
3. *Circle* the terms in the expression.
4. *Simplify* each term until it one number by evaluating each exponential and then multiplying and dividing from left to right.
5. *Combine* terms by adding and subtracting from left to right.

Example:

$$3(6-3\cdot 1) \quad + \quad 4\cdot 5^2 \quad - \quad \tfrac{13-3}{2}$$

$$3(6-3\cdot 1) \quad + \quad 4\cdot 5^2 \quad - \quad \tfrac{13-3}{2}$$

$$3(6-3\cdot 1) \quad + \quad 4\cdot 5^2 \quad - \quad \tfrac{13-3}{2}$$

$$3(3) \quad + \quad 4\cdot 5^2 \quad - \quad \tfrac{10}{2}$$

$$3(3) \quad + \quad 4\cdot 5^2 \quad - \quad \tfrac{10}{2}$$

$$9 \quad + \quad 4(25) \quad - \quad 5$$

$$9 \quad + \quad 100 \quad - \quad 5$$

$$109-5=104$$

Example 1: Evaluate $2x^2-3x+2$ for $x=-5$

Solution: $2(-5)^2-(-5)+2 \Rightarrow 2(25)-(-5)+2 \Rightarrow 50-(-5)+2 \Rightarrow 50+5+2=57$

Example 2: Evaluate $5\left(\frac{x+2y}{x-y}\right)$ for $x=-3$, $y=2$

Solution: $5\left(\frac{-3+2\cdot 2}{-3-2}\right) \Rightarrow 5\left(\frac{-3+4}{-3-2}\right) \Rightarrow 5\left(\frac{1}{-5}\right) \Rightarrow -1$

Now we can go back and solve the original problem.

a. $2x+3y+z \Rightarrow 2(-2)+3(-3)+5 \Rightarrow -4+-9+5=-8$

b. $x-y \Rightarrow (-2)-(-3) \Rightarrow -2+3=1$

c. $2\left(\frac{x+y}{z}\right) \Rightarrow 2\left(\frac{-2+-3}{5}\right) \Rightarrow 2\left(\frac{-5}{5}\right) \Rightarrow 2(-1)=-2$

d. $3x^2-2x+1 \Rightarrow 3(-2)^2-2(-2)+1 \Rightarrow 3(4)-2(-2)+1 \Rightarrow 12-(-4)+1=17$

e. $3y(x+x^2-y) \Rightarrow 3(-3)(-2+(-2)^2-(-3)) \Rightarrow 3(-3)\left(-2+4-(-3)\right) \Rightarrow 3(-3)(5)=-45$

f. $\frac{-z^2(1-2x)}{y-x} \Rightarrow \frac{-5^2(1-2(-2))}{-3-(-2)} \Rightarrow \frac{-25(1-(-4))}{-3-(-2)} \Rightarrow \frac{-25(5)}{-1}=125$

Here are some more to try. Evaluate each expression using $x=4,\ y=-2,\ z=-3$.

1. $2x-3$
2. z^2+5
3. $3z-2y$
4. $xy-4z$
5. $y-2+x$
6. $z-8-y$
7. $x^2+10x-20$
8. $2\left(\frac{2+x}{y+1}\right)$
9. y^2-3y+7
10. $2yz-x^2$
11. $-\frac{x}{3y}$
12. $(y+z)\cdot\frac{1}{4}x$
13. $2z(y+x^2-x)$
14. $\frac{10+y}{3y(x+1)}$
15. $2\left(\frac{x+y}{y}\right)$
16. $(2x+y^2)(3+z)$
17. $y-5+3z^2$
18. $x^2+12z-4y$
19. $\frac{2y^2x}{x+2}$
20. $x(3+zy)-2x^2$
21. $z^2+8zy-y^2$
22. x^3-4y
23. $6z-y^2+\frac{x+2}{z}$
24. $\frac{-y^2(xz-5y)}{3x-4y}$

Answers:

1. 5	2. 14
3. –5	4. 4
5. 0	6. –9
7. 36	8. –12
9. 17	10. –4
11. $\frac{2}{3}$	12. –5
13. –60	14. $-\frac{4}{15}$
15. –2	16. 0
17. 20	18. –12
19. $5\frac{1}{3}$	20. 4
21. 53	22. 72
23. –24	24. $\frac{2}{5}$

Checkpoint Number 3

Problem 3-126

Computations with Positive and Negative Fractions

Answers to problem 3-126: a. $-\frac{17}{24}$, b. $3\frac{5}{6}$, c. $1\frac{2}{5}$, d. $-1\frac{1}{3}$, e. $3\frac{7}{12}$, f. $2\frac{2}{7}$

Use the same processes with positive and negative fractions as are done with integers (positive and negative whole numbers.)

Example 1: Compute $\frac{1}{3}+\left(-\frac{9}{20}\right)$

Solution: When adding a positive number with a negative number, subtract the values and the number further from zero determines the sign.

$$\frac{1}{3}+-\frac{9}{20}=\frac{1}{3}\cdot\frac{20}{20}+-\frac{9}{20}\cdot\frac{3}{3}=\frac{20}{60}+-\frac{27}{60}=-\frac{7}{60}$$

Example 2: Compute $-1\frac{1}{4}-\left(-3\frac{9}{10}\right)$

Solution: Change any subtraction problem to "addition of the opposite" and then follow the addition process.

$$-1\frac{1}{4}-\left(-3\frac{9}{10}\right)\Rightarrow-1\frac{1}{4}+3\frac{9}{10}=-1\frac{5}{20}+3\frac{18}{20}=2\frac{13}{20}$$

Example 3: Compute $-1\frac{1}{4}\div 7\frac{1}{2}$

Solution: With multiplication or division, if the signs are the same, then the answer is positive. If the signs are different, then the answer is negative.

$$-1\frac{1}{4}\div 7\frac{1}{2}=-\frac{5}{4}\div\frac{15}{2}=-\frac{5}{4}\cdot\frac{2}{15}=-\frac{\cancel{5}\cdot\cancel{2}}{\cancel{2}\cdot 2\cdot 3\cdot\cancel{5}}=-\frac{1}{6}$$

Now we can go back and solve the original problem.

a. Both numbers are negative so add the values and the sign is negative.

$$-\frac{1}{3}+-\frac{3}{8}=-\frac{1}{3}\cdot\frac{8}{8}+-\frac{3}{8}\cdot\frac{3}{3}=-\frac{8}{24}+-\frac{9}{24}=-\frac{17}{24}$$

b. Change the subtraction to addition of the opposite.

$$2\frac{1}{3}-\left(-1\frac{1}{2}\right)=2\frac{1}{3}+1\frac{1}{2}=2+\frac{1}{3}\cdot\frac{2}{2}+1+\frac{1}{2}\cdot\frac{3}{3}=2+\frac{2}{6}+1+\frac{3}{6}=3\frac{5}{6}$$

c. The signs are the same so the product is positive. Multiply as usual.

$-4\,\frac{1}{5} \cdot -\frac{1}{3} = -\frac{21}{5} \cdot -\frac{1}{3} = \frac{7 \cdot \cancel{3} \cdot 1}{5 \cdot \cancel{3}} = \frac{7}{5} = 1\,\frac{2}{5}$

d. The signs are different so the quotient is negative. Divide as usual.

$\frac{2}{3} \div -\frac{1}{2} = \frac{2}{3} \cdot -\frac{2}{1} = -\frac{2 \cdot 2}{3 \cdot 1} = -\frac{4}{3} = -1\,\frac{1}{3}$

e. When adding a positive number with a negative number, subtract the values and the number further from zero determines the sign.

$-1\,\frac{3}{4} + 5\,\frac{1}{3} = -\frac{7}{4} + \frac{16}{3} = -\frac{7}{4} \cdot \frac{3}{3} + \frac{16}{3} \cdot \frac{4}{4} = -\frac{21}{12} + \frac{64}{12} = \frac{43}{12} = 3\,\frac{7}{12}$

f. The signs are the same so the quotient is positive. Divide as usual.

$-2\,\frac{2}{3} \div -1\,\frac{1}{6} = -\frac{8}{3} \div -\frac{7}{6} = -\frac{8}{3} \cdot -\frac{6}{7} = \frac{8 \cdot \cancel{3} \cdot 2}{\cancel{3} \cdot 7} = \frac{16}{7} = 2\,\frac{2}{7}$

Here are some more to try. Compute each of the following problems with fractions.

1. $-\frac{2}{3} + \frac{1}{2}$
2. $\frac{3}{4} - \left(-\frac{5}{12}\right)$
3. $-\frac{5}{7} + \frac{2}{3}$
4. $-1\,\frac{6}{7} + \left(-\frac{3}{4}\right)$
5. $\left(3\,\frac{1}{3}\right) \cdot \left(-\frac{2}{5}\right)$
6. $-2\,\frac{1}{4} \cdot \frac{2}{3}$
7. $-2\,\frac{7}{12} \div -\frac{1}{6}$
8. $3\,\frac{1}{2} + \left(-4\,\frac{3}{8}\right)$
9. $-1\,\frac{1}{4} - \left(-3\,\frac{1}{6}\right)$
10. $\left(2\,\frac{5}{9}\right) \cdot \left(-\frac{3}{7}\right)$
11. $-4\,\frac{3}{4} - \left(-\frac{5}{7}\right)$
12. $\frac{2}{3} \div -1\,\frac{4}{9}$
13. $-\frac{7}{9} \cdot 2\,\frac{3}{4}$
14. $-\frac{3}{5} \div -1\,\frac{1}{10}$
15. $-5\,\frac{1}{2} \div -\frac{3}{4}$
16. $10\,\frac{5}{8} + \left(-2\,\frac{1}{2}\right)$
17. $5\,\frac{1}{5} + \left(-2\,\frac{2}{15}\right)$
18. $12\,\frac{3}{4} - \left(-1\,\frac{5}{8}\right)$
19. $-2\,\frac{7}{9} \cdot 3\,\frac{1}{7}$
20. $-4\,\frac{1}{5} \div -\frac{3}{10}$
21. $5\,\frac{1}{12} - \left(-2\,\frac{6}{7}\right)$
22. $-6\,\frac{1}{7} \cdot -\frac{4}{5}$
23. $-2\,\frac{3}{8} \div 3\,\frac{1}{4}$
24. $-4\,\frac{3}{10} - 1\,\frac{1}{5}$

Answers:

1. $-\frac{1}{6}$	2. $1\frac{1}{6}$
3. $-\frac{1}{21}$	4. $-2\frac{17}{28}$
5. $-1\frac{1}{3}$	6. $-1\frac{1}{2}$
7. $15\frac{1}{2}$	8. $-\frac{7}{8}$
9. $1\frac{11}{12}$	10. $-1\frac{2}{21}$
11. $-4\frac{1}{28}$	12. $-\frac{6}{13}$
13. $-2\frac{5}{36}$	14. $\frac{6}{11}$
15. $7\frac{1}{3}$	16. $8\frac{1}{8}$
17. $3\frac{1}{15}$	18. $14\frac{3}{8}$
19. $-8\frac{46}{63}$	20. 14
21. $7\frac{79}{84}$	22. $4\frac{32}{35}$
23. $-\frac{19}{26}$	24. $-5\frac{1}{2}$

Checkpoint Number 4

Problem 5-76

Unit Rates and Proportions

Answers to problem 5-76: a. \$0.84, b. 0.6, c. $\approx \$3.50$, d. \$11.88, e. ≈ 169 shots, f. $3\frac{1}{3}$ teaspoons

A rate is a ratio comparing two quantities and a unit rate has a denominator of one after simplifying. Unit rates or proportions (ratio equations) may be used to solve ratio problems. Solutions may also be approximated by looking at graphs of lines passing through the origin and the given information.

Example 1: Sam paid \$4.95 for $\frac{3}{4}$ pound of his favorite trail-mix. What is the unit price (dollars per pound)?

Solution: The unit price is $\frac{\$4.95}{\frac{3}{4}\text{ pound}}$. To create a unit rate we need a denominator of "one."

$$\frac{\$4.95\cdot\frac{4}{3}}{\frac{3}{4}\text{ pound}\cdot\frac{4}{3}}=\frac{\$4.95\cdot\frac{4}{3}}{1\text{ pound}}=\$4.95\cdot\frac{4}{3}=\$6.60\text{ per pound}$$

Example 2: Based on the table, what is the unit growth rate (meters per year)?

+2

height (m)	15	17
years	75	85

+10

Solution:

$$\frac{2\text{ meters}}{10\text{ years}}=\frac{2\text{ meters}\div 10}{10\text{ years}\div 10}=\frac{\frac{1}{5}\text{ meter}}{1\text{ year}}=\frac{1}{5}\ \frac{\text{meter}}{\text{year}}$$

Note: This same kind of information might be determined by looking at a graph.

Example 3: In line at the movies are 146 people in front of you. If you count 9 tickets sold in 70 seconds, how long will it take before you buy your ticket?

Solution: The information may be organized in a proportion $\frac{9\text{ tickets}}{70\text{ seconds}}=\frac{146\text{ tickets}}{x}$. Solving the proportion $\frac{9}{70}=\frac{146}{x}$ yields $9x=10220$ so $x\approx 1135.56$ seconds or ≈ 19 minutes.

Now we can go back and solve the original problem.

a. $\frac{\$1.89}{2\frac{1}{4}\text{ pound}} = \frac{\$1.89 \cdot \frac{4}{9}}{\frac{9}{4}\text{ pound} \cdot \frac{4}{9}} = \frac{\$1.89 \cdot \frac{4}{9}}{1\text{ pound}} = \$1.89 \cdot \frac{4}{9} = \$0.84$ per pound

b. For every increase of three grams, the length increases by five centimeters.
$\frac{3\text{ grams}}{5\text{ cm}} = \frac{3}{5}\frac{\text{gram}}{\text{cm}}$

c. The graph passes through (10 bottles, \$35). $\frac{\$35}{10\text{ bottles}} = \$3.50$ per bottle.

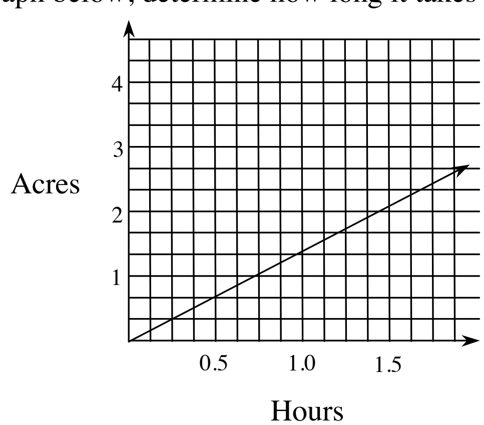
d. $\frac{200\text{ vitamins}}{\$4.75} = \frac{500\text{ vitamins}}{x} \Rightarrow 200x = 237.50 \Rightarrow x \approx \$11.88$

e. $\frac{72\text{ made}}{85\text{ attempts}} = \frac{x}{200\text{ attempts}} \Rightarrow 85x = 14400 \Rightarrow x \approx 169$ made

f. $\frac{\frac{1}{2}\text{teaspoon}}{\frac{3}{4}\text{ cup}} = \frac{x}{5\text{ cups}} \Rightarrow \frac{3}{4}x = \frac{5}{2} \Rightarrow x = \frac{10}{3} = 3\frac{1}{3}$ cups

Here are some more to try. In problems 1 through 10 find the unit rate and in problems 11 through 20 use ratios to solve each problem.

1. Amy usually swims 20 laps in 30 minutes. What is her rate in laps per minute?
2. For $\frac{3}{4}$ of a pound of peanuts, Jimmy paid \$2.35. What is the unit rate (dollars per pound)?
3. In 7 minutes, Jack types 511 words. How many words does he type per minute?
4. Using the graph below, determine how long it takes to mow an acre of grass.



5. The table below illustrates the growth of baby Martha through her first months. Based on the information given, how much did Martha grow each month?

Length (in.)	19	21.6	25.5
Age (months)	1	3	6

6. If Christy and Lucas eat 4 cups of popcorn in 30 minutes, how many minutes does it take them to eat a cup of popcorn?

7. In the past year, Ana has spent $7, 440 on her rent. What is the unit cost (dollars per month)?

8. While constructing his house, the little piggy found that 4 bundles of sticks contained a total of 208 sticks. How many sticks are in each bundle?

9. Meg knows that it takes 16 minutes to read $\frac{7}{8}$ of the chapter. How long will it take her to read the entire chapter?

10. At the frozen yogurt shop, Colin pays $3.28 for his treat, which weighs $8\frac{1}{5}$ ounces. What is the unit cost (cents per ounce)?

11. When Sarah does her math homework, she finishes 10 problems every 12 minutes. How long will it take for her to complete 35 problems?

12. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?

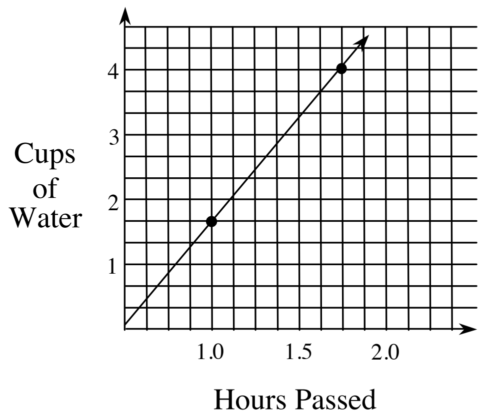
13. The tax on a $400 painting is $34. What should be the tax on a $700 painting?

14. Use the table below to write and solve a ratio for how long it will take Isadora to earn $120.

$ Earned	25	37.50	75
Days Worked	4	6	12

15. While baking, Hannah discovered a recipe that required $\frac{3}{4}$ cups of sugar for every $2\frac{1}{4}$ cups of flour. How many cups of sugar will she need for 4 cups of flour?

16. Based on the graph, how many cups of water does Shelby drink in 7 hours?



17. My brother grew $1\frac{3}{4}$ inches in $2\frac{1}{2}$ months. How much should he grow in one year?

18. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 90 minutes?

19. If Caitlin needs $1\frac{7}{8}$ cans of paint for each room in her house, how many cans of paint will she need to paint the 9-room house?

20. Stephen receives 6 gumballs for every two hours of homework he does. If he wants 22 gumballs, how many hours will he need to work?

Answers:

1. $\frac{2}{3}\ \frac{\text{laps}}{\text{minute}}$	2. \$3.13
3. $73\ \frac{\text{words}}{\text{minute}}$	4. $\frac{3}{4}$ hours
5. $1\frac{3}{10}\ \frac{\text{inches}}{\text{month}}$	6. $7\frac{1}{2}\ \frac{\text{minutes}}{\text{cup}}$
7. $\frac{\$620}{\text{month}}$	8. $52\ \frac{\text{sticks}}{\text{bundle}}$
9. $18\frac{2}{7}$ minutes	10. $\frac{\$0.40}{\text{ounce}}$
11. 42 minutes	12. $19\frac{1}{5}$ hours
13. \$59.50	14. $19\frac{1}{5}$ days
15. $1\frac{1}{3}$ cups	16. $11\frac{2}{3}$ cups
17. $8\frac{2}{5}$ inches	18. $8\frac{1}{28}$ miles
19. $16\frac{7}{8}$ cans	20. $7\frac{1}{3}$ hours

Checkpoint Number 5

Problem 6-111

Solving Equations

Answers to problem 6-111: a. -2, b. $1\frac{1}{2}$, c. 3, d. no solution

Equations may be solved in a variety of ways. Commonly, the first steps are to remove parenthesis using the distributive property and then simplify by combining like terms. Next isolate the variable on one side and the constants on the other. Finally, divide to find the value of the variable. Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, 2 = 4), there is <u>no solution</u> to the problem. When the result is the same expression or number on each side of the equation (for example, $x+3=x+3$) it means that <u>all numbers</u> are solutions.

Example 1: Solve $\mathbf{4x+4x-3=6x+9}$

Solution:

$$\begin{aligned} 4x+4x-3&=6x+9 && \text{problem}\\ 8x-3&=6x+9 && \text{simplify}\\ 2x&=12 && \text{add 3, subtract } 6x \text{ on each side}\\ x&=6 && \text{divide} \end{aligned}$$

Example 2: Solve $\mathbf{-4x+2-(-x+1)=-3+(-x+5)}$

Solution:

$$\begin{aligned} -4x+2-(-x+1)&=-3+(-x+5) && \text{problem}\\ -4x+2+x-1&=-3-x+5 && \text{remove parenthesis (flip)}\\ -3x+1&=-x+2 && \text{simplify}\\ -2x&=1 && \text{add } x\text{, subtract 1 to each side}\\ x&=-\tfrac{1}{2} && \text{divide} \end{aligned}$$

Now we can go back and solve the original problem.

a.
$$\begin{aligned} 3x+7&=-x-1\\ 4x&=-8\\ x&=-2 \end{aligned}$$

b.
$$\begin{aligned} 1-2x+5&=4x-3\\ -2x+6&=4x-3\\ 9&=6x\\ 1\tfrac{1}{2}&=x \end{aligned}$$

c.
$$\begin{aligned} -2x-6&=2-4x-(x-1)\\ -2x-6&=2-4x-x+1\\ -2x-6&=-5x+3\\ 3x&=9\\ x&=3 \end{aligned}$$

d.
$$\begin{aligned} 3x-4+1&=-2x-5+5x\\ 3x-3&=3x-5\\ -3&=-5\\ -3&\neq-5 \Rightarrow \text{no solution} \end{aligned}$$

Here are some more to try. Solve each equation.

1. $2x-3=-x+3$	2. $3x+2+x=x+5$
3. $6-x-3=4(x-2)$	4. $4x-2-2x=x-5$
5. $-(x+3)=2x-6$	6. $-x+2=x-5-3x$
7. $1+3x-x=x-4+2x$	8. $5x-3+2x=x+7+6x$
9. $4y-8-2y=4$	10. $-x+3=6$
11. $-2+3y=y-2-4y$	12. $2(x-2)+x=5$
13. $-x-3=2x-6$	14. $10=x+5+x$
15. $2x-1-1=x-3-(-5+x)$	16. $3+3x-x+2=3x+4$
17. $-4+3x-1=2x+1+2x$	18. $2x-7=-x-1$
19. $7=3x-4-(x+2)$	20. $5y+(-y-2)=4+y$

Answers:

1. 2	2. 1
3. $2\frac{1}{5}$	4. –3
5. 1	6. –7
7. 5	8. no solution
9. 6	10. –3
11. 0	12. 3
13. 1	14. $2\frac{1}{2}$
15. 2	16. 1
17. –6	18. 2
19. $6\frac{1}{2}$	20. 2

Checkpoint Number 6

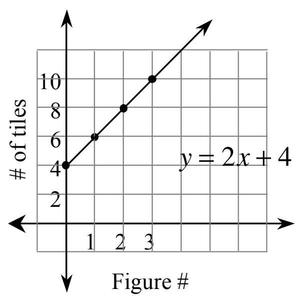
Problem 7-114

Multiple Representations

Answers to problem 7-114:

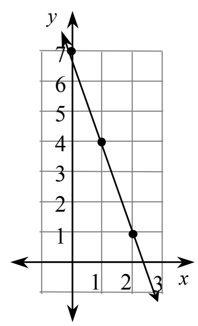
a.

figure #	0	1	2	
# of tiles	4	6	8	

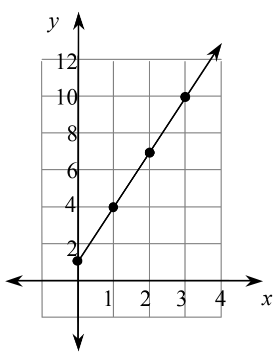


b.

x	0	1	2	
y	7	4	1	



c. $y = 3x + 1$

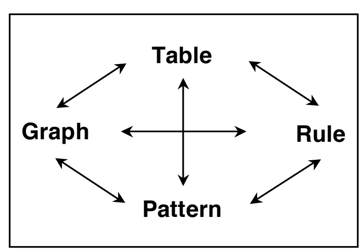


d.

figure #	0	1	2	3	
# of tiles	6	4	2	0	

$y = -2x + 6$

If you know one representation of a linear pattern, it can also be represented in three other different ways. Linear representations may be written as equations in the form $y = mx + b$ where m is the growth factor and b is the starting value.



Example 1: Using the pattern at right, create an $x \to y$ table, a graph, and write the rule as an equation.

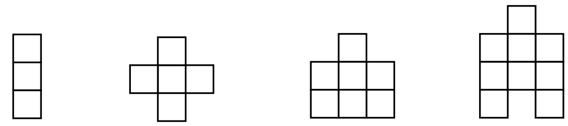
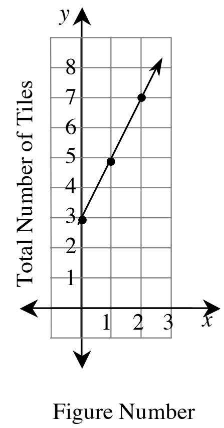


Figure 0 Figure 1 Figure 2 Figure 3

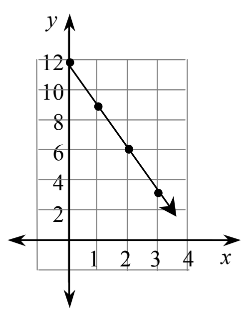
Solution: The number of tiles matched with the figure number gives the $x \to y$ table. Plotting those points gives the graph. Using the starting value for b and the growth factor for m gives the information to write the equation in $y = mx + b$ form.

figure # (x)	0	1	2	3
# of tiles (y)	3	5	7	9

The starting number is 3 tiles and the pattern grows by 2 tiles each figure so the equation is $y = 2x + 3$.



Example 2: Write an $x \to y$ table and the rule (equation) based on the graph below.



Solution:

Place the given points in a table:

x	0	1	2	3
y	12	9	6	3

The starting value is 12 and the "growth" factor is –3. The equation is:

$y = -3x + 12$

Now we can go back and solve the original problem.

a. Start by making a table from the tile pattern.

figure #	0	1	2	
# of tiles	4	6	8	

Since the starting value is 4 and the growth factor is 2 the equation is $y = 2x + 4$.

b. Make a table by replacing x in the rule with $0, 1, 2, 3$ and computing y. See the table above. Plot the points from the table to get the graph shown above.

Looking at the table, the Plot the points from the table to get the graph shown above.

c. Looking at the table, the y-value changes from 1 to 10 in three growth factors so each one must be three. Since the starting value is 1 and the growth factor is 3, the table and equation are: See the graph above.

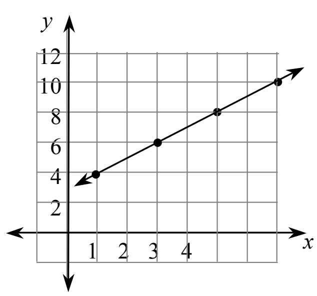
x	0	1	2	3	
y	1	4	7	10	

$y = 3x + 1$

d. Again the table may be determined from the points on the graph. Looking at the table, the starting value is 6 and the growth factor is –2 so the equation is $y = -2x + 6$.

Here are some more to try. For each situation, complete the multiple representations web by finding the missing $x \rightarrow y$ table, graph, and/or rule. Since there are many possible patterns, it is not necessary to create one.

1.



2.

x	0			3	
y	2			11	

3. $y = 3x - 2$

4.

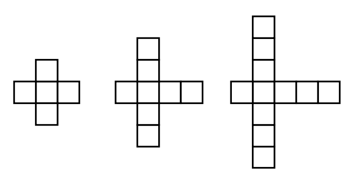
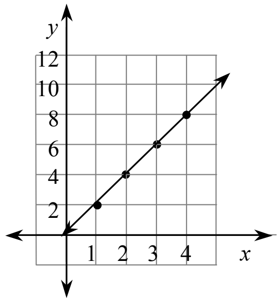


Figure 1 Figure 2 Figure 3

5.

x	1		4		
y	1.5		3		

6. $y = -4x + 12$

7. 

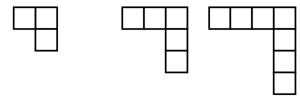
8. 

Figure 1   Figure 2   Figure 3

9. $y = -2x + 7$

10.

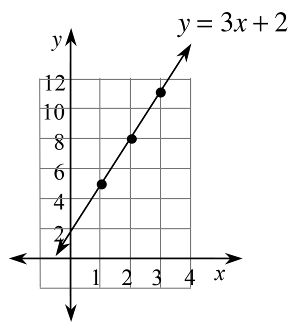
x	-1		2	
y	$4\frac{1}{4}$		$6\frac{1}{2}$	

Answers:

1.

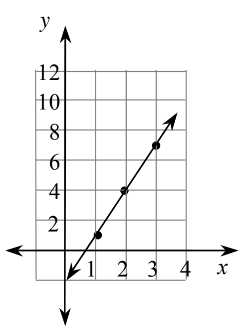
x	1	2	3	4
y	4	5	6	7

$y = x + 3$

2. 

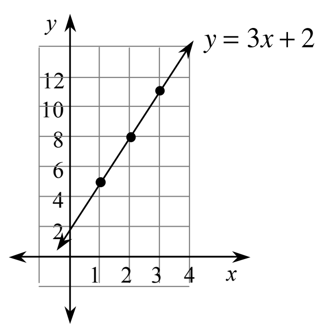
3.

x	0	1	2	3
y	-2	1	4	7

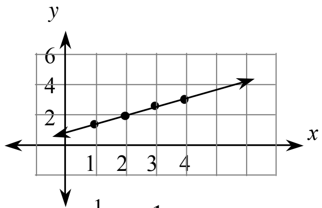


4.

x	0	1	2	3
y	2	5	8	11



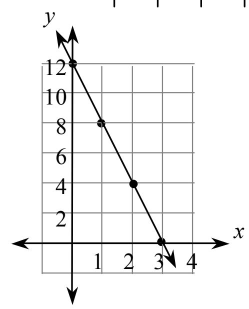
5.



$y = \frac{1}{2}x + 1$

6.

x	0	1	2	3
y	12	8	4	0



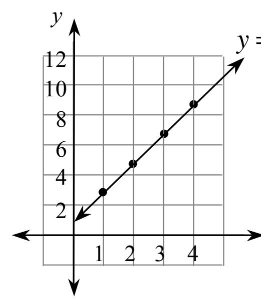
7.

x	0	1	2	3
y	0	2	4	6

$y = 2x$

8.

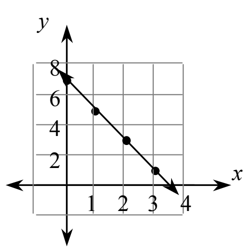
x	0	1	2	3
y	1	3	5	7



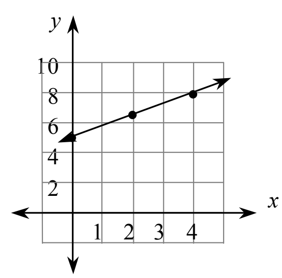
$y = 2x + 1$

9.

x	0	1	2	3
y	7	5	3	1



10.



$y = \frac{3}{4}x + 5$

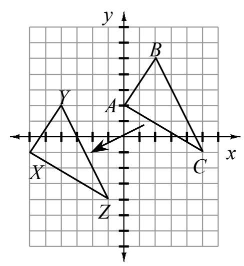
Checkpoint Number 7

Problem 8-64

Transformations

Answers to problem 8-64: (Given in the order *X, Y, Z*) a. (–2, 2), (–2, 0), (1, 2), b. $(4,-1),(4,-3),(1,-1)$, c. $(-1,4),(-3,4),(-1,1)$, d. $(-8,-2),(-8,6),(-2,2)$

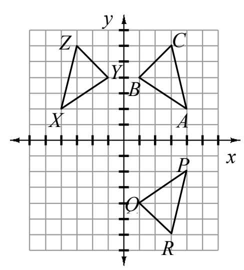
Rigid transformations are ways to move an object while not changing its shape or size. A translation (slide) moves an object vertically, horizontally or both. A reflection (flip) moves an object across a line of reflection as in a mirror. A rotation (turn) moves an object clockwise or counterclockwise around a point. A dilation is a non-rigid transformation. It produces a similar figure to the original by proportionally shrinking or stretching the figure from a point.



Example 1: Translate (slide) $\triangle ABC$ left six units and down three units. Give the coordinates of the new $\triangle XYZ$.

Solution: The original vertices are *A* (0, 2), *B* (2, 5), and *C* (5, –1). The new vertices are *X* (–6, –1), *Y* (–4, 2), and *Z* (–1, –4).

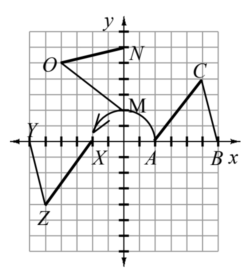
Example 2: Reflect (flip) $\triangle ABC$ across the *x*-axis to get $\triangle PQR$. Give the coordinates of the new $\triangle PQR$.



Now reflect (flip) $\triangle ABC$ across the *y*-axis to get $\triangle XYZ$. Give the coordinates of the new $\triangle XYZ$.

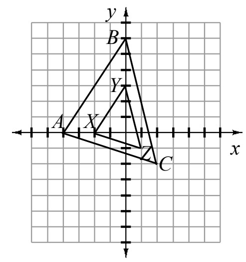
Solution: The key is that the reflection is the same distance from the axis as the original figure. For the first reflection the points are $P\ (4,-2)$, $Q\ (1,-4)$, and $R\ (3,-6)$. For the second reflection the points are $X\ (-4,2)$, $Y\ (-1,4)$, and $Z\ (-3,6)$.

Example 3: Rotate (turn) $\triangle ABC$ counterclockwise 90° about the origin to get $\triangle MNO$. Give the coordinates of the new $\triangle MNO$. Then rotate $\triangle MNO$ counterclockwise another 90° to get $\triangle XYZ$. Give the coordinates of the new $\triangle XYZ$.



Solution: After the first 90° rotation, the coordinates of *A* (2,0), *B* (6,0), and *C* (5, 4) became *M* (0, 2), $N\ (0,6)$, and *O* (–4, 5). Note that each original point $(x,\ y)$ became $(-y,\ x)$. After the next 90° rotation, the coordinates of the vertices are now $X\ (-2,0)$, *Y* (–6, 0), and *Z* (–5, –4). After the 180° rotation each of the points $(x,\ y)$ in the original $\triangle ABC$ became $(-x,-y)$. Similarly a 270° counterclockwise rotation or a 90° clockwise rotation about the origin takes each point (x,y) to the point $(y,-x)$.

Example 4: Dilate (enlarge/reduce) $\triangle ABC$ by a scale factor of $\frac{1}{2}$ from the origin to get $\triangle XYZ$. Give the coordinates of the new $\triangle XYZ$.



Solution: Multiplying the coordinates of the vertices of $\triangle ABC$ by the scale factor gives the coordinates of the vertices of the new $\triangle XYZ$. They are $X\ (-2, 0)$, $Y\ (0, 3)$ and $Z\ (1, -1)$.

Now we can go back and solve the original problem.

a. Sliding the triangle two units to the right and three units up increases the x-values by 2 and the y-values by 3. See the answers above.

b. Reflecting the triangle across the y-axis changes the x-values to the opposite and keeps the y-values the same. See the answers above.

c. Rotating the triangle 90° clockwise moves the triangle into the second quadrant. The original y-value becomes the new x-value and the opposite of the original x-value becomes the y-value. See the answers above.

d. Dilating the triangle by a scale factor of two multiplies all of the coordinates by two. See the answers above.

Here some more to try. For the following problems, refer to the figures below:

Figure A

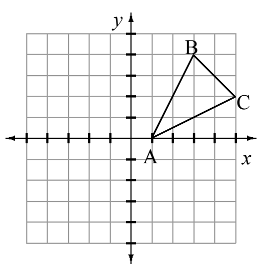


Figure B

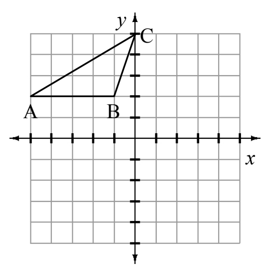
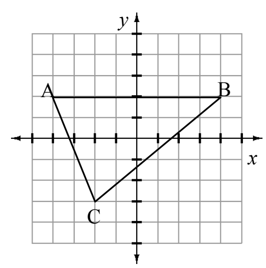


Figure C



State the new coordinates after each transformation.

1. Slide figure A left 2 units and down 2 units.
2. Rotate figure C 180° counterclockwise about the origin.
3. Flip figure B across the y-axis.
4. Flip figure B across the x-axis.
5. Slide figure A right 4 units and up 1 unit.
6. Flip figure C across the y-axis.

7. Rotate figure B 270° counterclockwise about the origin.
8. Slide figure C left 1 unit and up 2 units.
9. Rotate figure A 90° clockwise about the origin.
10. Flip figure B across the line $y = 3$.
11. Dilate figure B by a scale factor of 3.
12. Rotate figure A 180° counterclockwise about the origin.
13. Slide figure B 4 units down and 4 units to the right.
14. Dilate figure A by a scale factor of 4.
15. Rotate figure C 90° clockwise about the origin.
16. Flip figure A across the y-axis.
17. Dilate figure C by a scale factor of $\frac{1}{2}$.
18. Slide figure C 3 units right and 2 units down.
19. Rotate figure C 180° about the origin clockwise.
20. Dilate figure A by a scale factor of 2.

Answers:

1. (–1, –2), (1, 2), (3,0)
2. (4, –2), (–4, –2), (2, 3)
3. (5, 2), (1, 2), (0, 5)
4. (–5, –2), (–1, –2), (0, –5)
5. (5, 1), (7, 5), (9, 3)
6. (4, 2), (–4, 2), (2, –3)
7. (2, 6), (2, 1), (5, 0)
8. (–5, 4), (3, 4), (–3, –1)
9. (0, –1), (4, –3), (2, –5)
10. (5, 4), (1, 4), (0, 1)
11. (–15, 6), (–3, 6), (0, 15)
12. (–1, 0), (–3, –4), (–5, –2)
13. (–1, –2), (3, –2), (4, 1)
14. (4, 0), (12, 16), (20, 8)
15. (2, 4), (2, –4), (–3, 2)
16. (–1, 0), (–3, 4), (–5, 2)
17. (–2, 1), (2, 1), (–1, –1.5)
18. (–1, 0), (7, 0), (1, –5)
19. (4, –2), (–4, –2), (2, 3)
20. (2, 0), (6, 8), (10, 4)

Checkpoint Number 8

Problem 8-140

Solving Equations with Fractions (Fraction Busters)

Answers to problem 8-140: a. $\frac{15}{4} = 3\frac{3}{4}$, b. $\approx \$1.74$, c. -5, d. $(12, 16)$

Equations are usually easier to solve if there are no fractions. To eliminate fractions from an equation, multiply both sides of the equation by the common denominator.

Example 1: Solve $\frac{1}{2}x + \frac{2}{3} = \frac{3}{4}$

Solution: Start by multiplying both sides of the equation by 12 (the common denominator.) Simplify and then solve as usual.

$$\begin{aligned} 12\left(\tfrac{1}{2}x + \tfrac{2}{3}\right) &= 12\left(\tfrac{3}{4}\right) \\ 6x + 8 &= 9 \\ 6x &= 1 \\ x &= \tfrac{1}{6} \end{aligned}$$

Example 2: Solve $1.1x + 0.35x = 29$

Solution: Two decimal places means that the common denominator is 100. Multiply both sides by 100 and solve as usual.

$$\begin{aligned} 100(1.1x + 0.35x) &= 100(29) \\ 110x + 35x &= 2900 \\ 145x &= 2900 \\ x &= 20 \end{aligned}$$

Now we can go back and solve the original problem.

a.
$$\begin{aligned} \tfrac{1}{5}x + \tfrac{1}{3}x &= 2 \\ 15\left(\tfrac{1}{5}x + \tfrac{1}{3}x\right) &= 15(2) \\ 3x + 5x &= 30 \\ 8x &= 30 \\ x &= \tfrac{30}{8} = \tfrac{15}{4} = 3\tfrac{3}{4} \end{aligned}$$

b.
$$\begin{aligned} x + 0.15x &= \$2 \\ 100(x + 0.15x) &= 100(\$2) \\ 100x + 15x &= \$200 \\ 115x &= \$200 \\ x &\approx \$1.74 \end{aligned}$$

c.
$$\begin{aligned} \tfrac{x+2}{3} &= \tfrac{x-2}{7} \\ 21\left(\tfrac{x+2}{3}\right) &= 21\left(\tfrac{x-2}{7}\right) \\ 7(x+2) &= 3(x-2) \\ 7x + 14 &= 3x - 6 \\ 4x &= -20 \\ x &= -5 \end{aligned}$$

d.
$$\begin{aligned} y &= \tfrac{2}{3}x + 8 \\ y &= \tfrac{1}{2}x + 10 \\ \tfrac{2}{3}x + 8 &= \tfrac{1}{2}x + 10 \\ 6\left(\tfrac{2}{3}x + 8\right) &= 6\left(\tfrac{1}{2}x + 10\right) \\ 4x + 48 &= 3x + 60 \\ x &= 12 \\ y &= \tfrac{2}{3}(12) + 8 = 16 \\ &(12, 16) \end{aligned}$$

Here are some more to try. Solve each equation or system of equations.

1. $\frac{1}{6}x + \frac{2}{3}x = 5$

2. $y = 32x + 16$
   $y = 80x + 4$

3. $\frac{6}{15} = \frac{x-2}{40}$

4. $y = \frac{x}{3}$
   $y = \frac{4}{3}x - 9$

5. $\frac{x}{2} - 4 = \frac{x}{3}$

6. $\frac{x}{4} - 3 = \frac{x+4}{6} - 2$

7. $\frac{x}{10} + \frac{5}{12} = 3x - 1$

8. $\frac{2x-2}{6} - \frac{1}{2} = \frac{x}{2} - 2$

9. $0.2x + x = 30$

10. $y = \frac{x}{6} + \frac{1}{4}$
    $y = x - \frac{9}{4}$

11. $y = -3x + 2$
    $y = -\frac{15}{4}x + 3$

12. $x + 3\frac{2}{3} = 2x + \frac{1}{3}$

13. $\frac{x}{2} + \frac{4x}{3} = 2x - 1.5$

14. $\frac{x+1}{4x} = \frac{5}{16}$

15. $y = \frac{1}{3}x + 8$
    $y = -\frac{1}{2}x - 2$

16. $y = 7x + 2$
    $y = 2x - 10\frac{1}{2}$

17. $y = \frac{x}{2} - 1$
    $y = \frac{x+3}{12}$

18. $y = \frac{3}{2}x - 10$
    $y = \frac{x}{3} - \frac{x}{2}$

19. $\frac{x-1}{4} = \frac{7}{8}$

20. $\frac{2}{3}x = x - \frac{10}{3}$

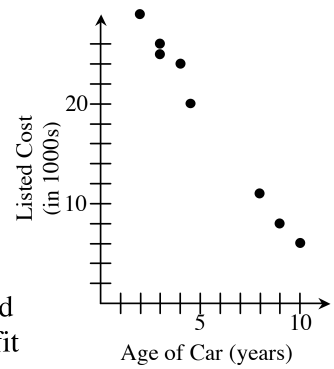
Answers:

1. $x = 6$
2. $x = \frac{1}{4}$, $y = 24$
3. $x = 18$
4. $x = 9$, $y = 3$
5. $x = 24$
6. $x = 20$
7. $x = \frac{5}{6}$
8. $x = 7$
9. $x = 25$
10. $x = 3$, $y = \frac{3}{4}$
11. $x = \frac{4}{3}$, $y = -2$
12. $x = 3\frac{1}{3}$
13. $x = 9$
14. $x = 4$
15. $x = -12$, $y = 4$
16. $x = \frac{1}{2}$, $y = 5\frac{1}{2}$
17. $x = 3$, $y = \frac{1}{2}$
18. $x = 6$, $y = -1$
19. $x = 4.5$
20. $x = 10$

Checkpoint Number 9

Problem 9-130

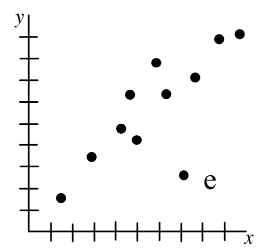
Scatter Plots and Association



Answers to problem 9-130: a. box plot, b. scatterplot, c. See graph at right, d. strong negative association, e. $y=-3x+35$, f. \$17,000, g. A slope of –3 means the car is losing \$3000 in value each year, y-intercept of 35 means the cost when new was \$35,000.

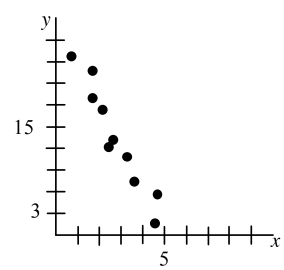
An association (relationship) between two numerical variables on a graph can be described by its form, direction, strength, and outliers. When the association has a linear form, a line a best fit can be drawn and its equation can be used to make predictions about other data.

Example 1: Describe the association in the scatterplot at right.



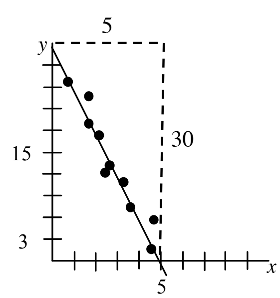
Solution: Looking from left to right, except for point (e), the points are fairly linear and increasing. This is a moderate, positive linear association. The point (e) is an outlier

Example 2: For the scatterplot, draw a line of best fit and determine the equation of the line.



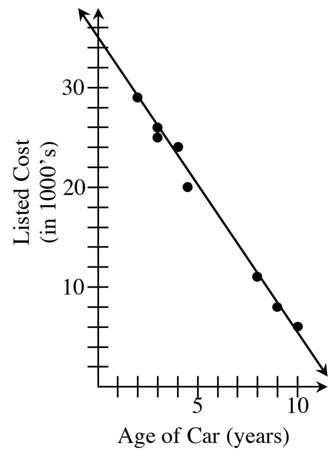
Solution: Use a ruler or straightedge to draw the line that approximates the trend of the points. If it is not a perfect line, approximately the same number of points will be above and below the line of best fit.

To determine the equation of the line, draw in a slope triangle and determine the ratio of vertical side to horizontal side. In this case it is $\frac{-30}{5}=-6$. Estimate the y-intercept by looking at where the line intersects the y-axis. In this case, it is approximately 30. The equation of any non-vertical line may be written in the form $y=mx+b$ where m is the slope and b is the y-intercept.



$y=-6x+30$

Now we can go back and solve the original problem.



a. Since the costs are a single set of data, a box plot is convenient to show the distribution.

b. Age and cost are two sets of related data so a scatterplot is appropriate.

c. Reading from left to right, the scatterplot is decreasing, linear and the points are close to the line of best fit. This is a strong, linear, negative association.

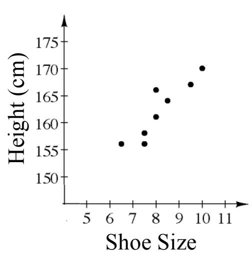
d. Looking at the line of best fit, the slope triangle has a ratio of $-\frac{3}{1}$ and the y-intercept is approximately 35. Placing that information into the equation of a line $y = mx + b$ yields $y = -3x + 35$.

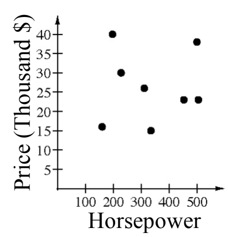
e. Substituting $x = 6$ into the equation of part (c) yields $y = -3(6) + 35 = 17$.

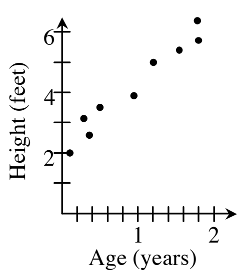
f. Slope represents the rate of change. A rate of change of –3 means that the value is decreasing by 3 units (in this case each unit is $1000) per year. The y-intercept represents the value at year zero or a new car.

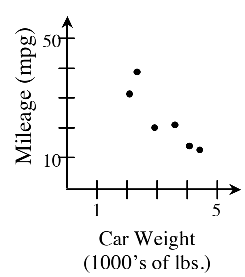
Here are some more to try.

In problems 1 through 4, describe the association.

1. 

2. 

3. 

4. 

In problems 5 through 8 plot the data, draw a line of best fit and approximate the equation of the line.

5.

Distance to airport	5	10	15	20	25	30
Cost of shuttle ($)	14	17	21	31	33	40

6.

Hours of exercise/month	3	6	9	12	15	18
Rate of heart attack/1000	24	21	18	12	6	0

7.

Hours spent studying	0	2	2.5	2.8	3	4.5	5
Score on test	65	70	70	85	80	95	100

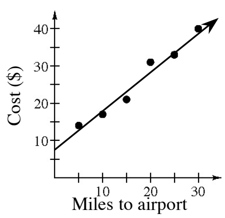
8.

Hours since purchase	0	2	4	6	8	10
Number of cookies	24	20	14	11	5	0

Answers:

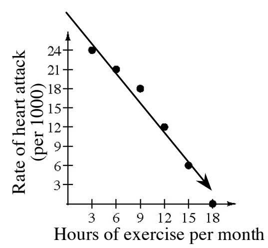
1. strong positive association
2. no association
3. strong positive association
4. strong negative association

5.



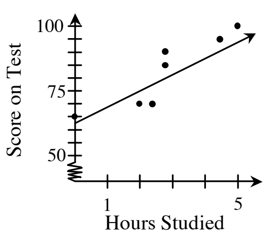
$\approx y = 1x + 8$

6.



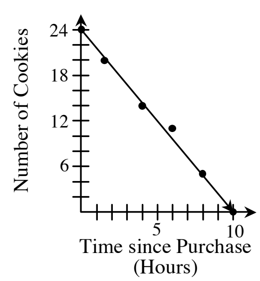
$\approx y = -1.5x + 30$

7.



$\approx y = 6x + 63$

8.



$\approx y = 2.4x + 10$

Checkpoint Number 10

Problem 10-29

Laws of Exponents and Scientific Notation

Answers to problem 10-29: a. 4^7, b. 1, c. $3^{-2} = \frac{1}{9}$, d. $\frac{49}{10} = 4.9$, e. 1.28×10^4,
f. 8×10^{-3}

The laws of exponents summarize several rules for simplifying expressions that have exponents. The rules are true for any base if $x \neq 0$.

$$x^a \cdot x^b = x^{(a+b)} \qquad (x^a)^b = x^{ab} \qquad \frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1 \qquad x^{-a} = \frac{1}{x^a}$$

Scientific notation is a way of writing a number as a product of two factors separated by a multiplication sign. The first factor must be less than 10 and greater than or equal to 1. The second factor has a base of 10 and an integer exponent.

Example 1: Simplify $4^2 \cdot 4^{-4}$

Solution: In a multiplication problem, if the bases are the same, add the exponents and keep the base. If the answer ends with a negative exponent, take the reciprocal and change the exponent to positive.

$$4^2 \cdot 4^{-4} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Example 2: Simplify $\frac{(3^2)^3 \cdot 5^4}{3^{-2} \cdot 5}$

Solution: Separate the fraction into two fractions with bases 3 and 5. With an exponent on an exponent, multiply the exponents. Next, to divide expressions with exponents and the same base, subtract the exponents.

$$\frac{(3^2)^3 \cdot 5^4}{3^{-2} \cdot 5} = \frac{(3^2)^3}{3^{-2}} \cdot \frac{5^4}{5} = \frac{3^6}{3^{-2}} \cdot \frac{5^4}{5^1} = 3^8 \cdot 5^3$$

The number in standard form is very large so it is common to give the answer using exponents.

Example 3: Multiply and give the answer in scientific notation. $(8\times10^4)\cdot(4.5\times10^{-2})$

Solution: Separate the number parts and the exponent parts. Multiply the number parts normally and the exponent part by adding the exponents. If this answer is not in scientific notation, change it appropriately.

$$(8\times10^4)\cdot(4.5\times10^{-2})=(8\times4.5)\cdot(10^4\times10^{-2})=36\times10^2=(3.6\times10^1)\times10^2=3.6\times10^3$$

Now we can be back and solve the original problem.

a. $4^2\cdot4^5=4^{(2+5)}=4^7$

b. $(5^0)^3=5^{0\cdot3}=5^0=1$

c. $3^{-5}\cdot3^3=3^{(-5+3)}=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

d. $10^{-1}\cdot7^2=\frac{1}{10}\cdot49=\frac{49}{10}=4.9$

e. $(8\times10^5)\cdot(1.6\times10^{-2})=12.8\times10^3=(1.28\times10^1)\times10^3=1.28\times10^4$

f. $\frac{4\times10^3}{5\times10^5}=\frac{4}{5}\times10^{(3-5)}=0.8\times10^{-2}=(8\times10^{-1})\times10^{-2}=8\times10^{-3}$

Here are some more to try. Simplify each expression. In problems 19 through 24 write the final answer in scientific notation.

1. $5^4\cdot5^{-1}$
2. $3^3\cdot3^3\cdot3^6$
3. $7^2\cdot(7^4)^{-2}$
4. $2^{-2}\cdot\frac{1}{2^2}\cdot2^3$
5. $3^3\cdot3^5\cdot\left(\frac{1}{3}\right)^2$
6. $(3^3\cdot4^{-6})^2\cdot6^7$
7. $3^{-3}\cdot3^0$
8. $19^1\cdot\frac{19}{19^3}\cdot\frac{2^{-2}}{2}$
9. $\frac{7^4\cdot9^2}{9^3\cdot7^2}$
10. $\frac{14^3}{14^{-2}}\cdot14^0$
11. $\left(\frac{2^2\cdot3^4}{3^3\cdot2^4}\right)^0$
12. $\frac{(3^2)^3}{3^6}\cdot3^4$
13. $\left(\frac{5^2}{5^4}\right)^{-1}$
14. $(7^2\cdot7^3)^4$
15. $\frac{3^3\cdot3^2\cdot3^{-3}}{3^{-4}\cdot3^3}$
16. $\left(\frac{1}{2^4}\right)^{-2}\cdot2\cdot2^0$

17. $\left(\frac{2^4}{7^{-3}}\right)\left(\frac{7^{-2}}{2^5}\right)^{-1}$

18. $\frac{9^3 \cdot 9^{-5}}{9^0}$

19. $(4.25 \times 10^3) \cdot (2 \times 10^5)$

20. $(1.2 \times 10^4) \cdot (7.1 \times 10^{-2})$

21. $(6.9 \times 10^7) \cdot (3 \times 10^2)$

22. $(5.63 \times 10^{-6}) \cdot (4 \times 10^{-7})$

23. $(6 \times 10^{-3})^2$

24. $2.7 \times 10^4 \div 3.2 \times 10^{-2}$

Answers:

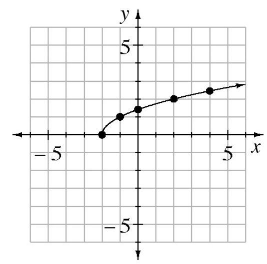
1. 5^3
2. 3^{12}
3. 7^{-6}
4. $2^{-1} = \frac{1}{2}$
5. 3^6
6. $3^{13} \cdot 2^{-17}$
7. $3^{-3} = \frac{1}{27}$
8. $19^{-1} \cdot 2^{-3} = \frac{1}{19 \cdot 2^3}$
9. $\frac{7^2}{9}$
10. 14^5
11. 1
12. 3^4
13. 5^2
14. 7^{10}
15. 3^3
16. 2^9
17. $2^9 \cdot 7^5$
18. 9^{-2}
19. 8.5×10^8
20. 8.52×10^2
21. 2.07×10^{10}
22. 2.252×10^{-12}
23. 3.6×10^{-5}
24. 8.4375×10^5

Checkpoint Number 11

Problem 10-88

Problems Involving Square Roots

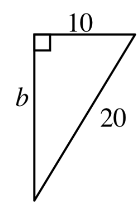
Answers to problem 10-88: a. $\sqrt{189} \approx 13.75$, b. $\sqrt{145} \approx 12.0$, c. ≈ 4.37 cm, d. See graph at right.



If the lengths of any two sides of a right triangle are known, the third side may be found using the Pythagorean theorem: $(\text{leg}\#1)^2 + (\text{leg}\#2)^2 = \text{hypotenuse}^2$.

Equations of the form $(\text{variable})^2 = \text{number}$ may be solved using square roots.

Example 1: Determine the length of the missing side in the triangle at right.



Solution: The hypotenuse (20) is the side opposite the right angle and the legs are 10 and *b*. Then substituting this information into the Pythagorean theorem:

$$10^2 + b^2 = 20^2$$
$$100 + b^2 = 400$$
$$b^2 = 300 \Rightarrow b = \sqrt{300} \approx 17.32 \text{ units}$$

Example 2: The distance (*d*) in meters an object falls under gravity in (*t*) seconds is found using the formula $d = 4.9t^2$. How long does it take an object to fall 100 meters?

Solution: Substitute into the formula and solve for *t* by first dividing to undo the multiplication by 4.9 and then square rooting to undo the squaring.

$$100 = 4.9t^2$$
$$\frac{100}{4.9} = t^2 \Rightarrow t = \sqrt{\frac{100}{4.9}} \approx 4.52 \text{ seconds}$$

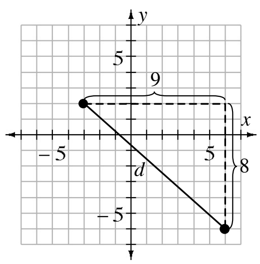
Now we can go back and solve the original problem.

a. Substituting the given information into the Pythagorean theorem:

$$10^2 + x^2 = 17^2$$

$$100 + x^2 = 289 \Rightarrow x^2 = 189 \Rightarrow x = \sqrt{189} \approx 13.75$$

b. Graph the two points and draw the segment (d) determined by them. Use that segment as the hypotenuse of a right triangle and determine the length of the legs. Use that information in the Pythagorean theorem to determine the distance.



$$9^2 + 8^2 = d^2$$

$$81 + 64 = d^2 \Rightarrow d^2 = 145 \Rightarrow d = \sqrt{145} \approx 12.0$$

c. Use the formula for area of a circle: $A = \pi r^2$, substitute the value of A, and solve for r.

$$60 = \pi r^2$$

$$\frac{60}{\pi} = r^2 \Rightarrow r = \sqrt{\frac{60}{\pi}} \approx 4.37 \text{ cm}$$

d. Create an $x \rightarrow y$ table, fill in values for x, and compute the values of y. Note that for all $x < -2$, y is a square root of a negative number, which is not a real number.

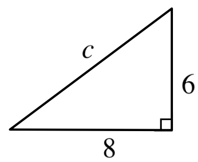
x	–3	–2	–1	0	1	2
y	E	0	1	1.4	1.7	2

See graph above.

Here are some more to try. Solve each problem.

In problems 1 through 6 find the length of the missing side.

1.



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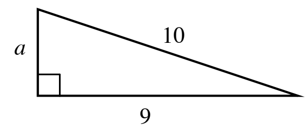

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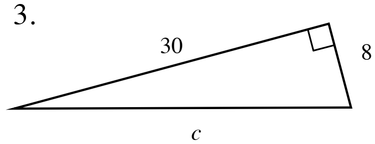

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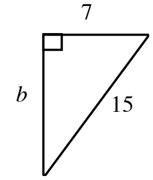
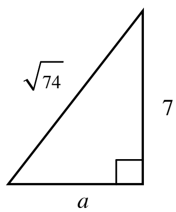
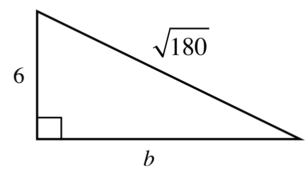


5.



6.



In problems 7 through 12 find the distance between the two points.

7. (1,2) and (7, 10)

8. (–2, 4) and (6, –1)

9. (–1, –3) and (3, 1)

10. (4, 3) and (–5, 12)

In problems 11 through 16 find the required information.

11. A circle has area 100 ft^2, what is the radius?

12. A ball falls from a second story window to the ground, a total of 15 feet. How long did it take?

13. $x^2 + 7 = 11$

14. $3x^2 - 8 = 40$

15. If a circle has an area of 75 square meters, what is the diameter?

16. A parachutist falls 300 feet before opening her parachute. How long was she in free fall?

In problems 17 through 20 make an $x \to y$ table and draw the graph.

17. $y = \sqrt{x} - 2$

18. $y = \sqrt{2x} + 3$

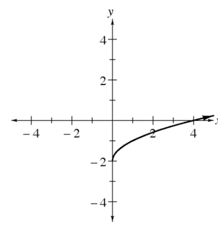
19. $y = \frac{\sqrt{x+3}}{2}$

20. $y = 2\sqrt{x-1}$

Answers:

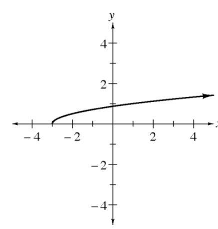
1. $c = 10$
2. $a = \sqrt{19} \approx 4.36$
3. $c = \sqrt{964} \approx 31.05$
4. $b = \sqrt{176} \approx 13.27$
5. $a = 5$
6. $b = 12$
7. 10
8. $\sqrt{89} \approx 9.43$
9. $\sqrt{32} \approx 5.66$
10. $\sqrt{162} \approx 12.73$
11. About 5.64 ft
12. About 1.75 seconds
13. $x = 2$
14. $x = 4$
15. About 4.89 meters
16. About 7.82 seconds
17.

x	0	1	2	3	4	5	6
y	–2	–1	–0.59	–0.27	0	0.24	0.45



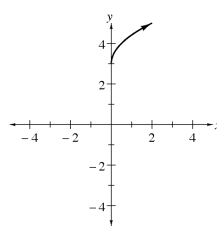
18.

x	–3	–2	–1	0	1	2	3
y	0	0.5	0.71	0.87	1	1.12	1.22



19.

x	0	1	2	3	4	5	6
y	3	4.41	5	5.45	5.83	6.16	6.46



20.

x	1	2	3	4	5	6
y	0	2	2.83	3.46	4	4.47